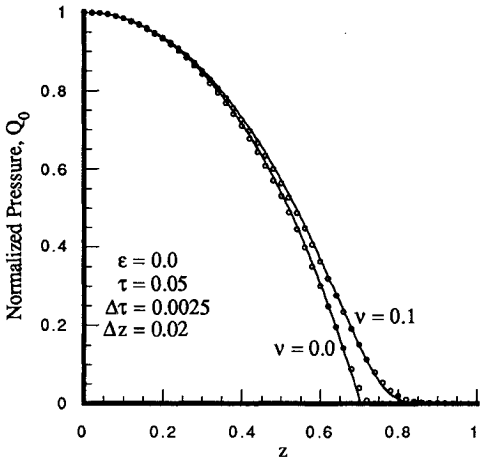

Fig. 5 Comparison of finite difference prediction to exact $\mathcal{O}(1)$ solution at $\tau = 0.05$.

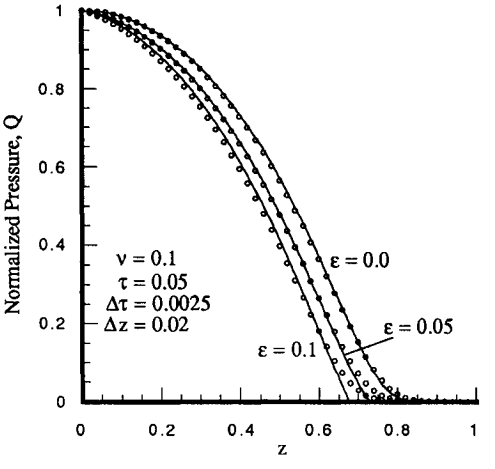

Fig. 6 Comparison of finite difference prediction to the perturbation solution for various values of small parameter ε at $\tau = 0.05$ and $\nu = 0.1$.

20 uniform time steps. An early time of $\tau = 0.05$ was chosen to confine the solution to the region $0 \leq z \leq 1.0$.

Figure 5 compares the exact $\mathcal{O}(1)$ solution to the finite difference prediction. For this figure, the deviation between the computational points and the exact solution is solely due to the finite difference approximation. Note that for the case of an initial vacuum ($\nu = 0$), the largest discrepancy occurs at the wave front. Because the finite difference equations are diffusive in nature, the case of a finite initial pressure ($\nu = 0.1$) is handled at the wave front more accurately than the wavelike initial vacuum case.

On the other hand, the $\mathcal{O}(\varepsilon)$ perturbation solution is generally satisfactory except at the wave front. As ε becomes large, the additive nature of the perturbation approximation removes the diffusive tail and causes the $\mathcal{O}(\varepsilon)$ solution to incorrectly assume a sharp wave front. Figure 6 shows that the finite difference approximation follows the $\mathcal{O}(\varepsilon)$ solution well until the wave front is reached. Additional refinement of the finite difference mesh and time step changes the finite difference solution somewhat, but does not come close to matching the ($\varepsilon = 0.1$) perturbation solution at the wave front.

Conclusions

A similarity solution is presented for transient compressible isothermal flow through porous media in which the permeability varies inversely with distance. A modification of the exact $\mathcal{O}(1)$ solution was derived by a regular perturbation method for the case of a finite permeability at the surface.

The finite difference approximation was used to identify a weakness in the perturbation correction at the wave front when both the initial pressure and the small parameter (ε) become large and significant. In all other aspects, the finite difference prediction agreed quite well with the perturbation solution.

Acknowledgment

Financial support of this work under NASA Grant NAG8-862 is gratefully acknowledged.

References

[1]Muskat, M., "The Flow of Compressible Fluids Through Porous Media and Some Problems in Heat Conduction," *Physics*, Vol. 5, March, 1934, pp. 71–94.

[2]Morrison, F. A., "Transient Gas Flow in a Porous Column," *Industrial Engineering Chemistry Fundamentals*, Vol. 11, No. 2, 1972, pp. 191–197.

[3]Pascal, H., "On Non-Linear Effects in Unsteady Flows Through Porous Media," *International Journal of Non-Linear Mechanics*, Vol. 26, No. 2, 1991, pp. 251–261.

[4]Pantankar, S. V., *Numerical Heat Transfer and Fluid Flow*, 1st ed., Hemisphere, New York, 1980, Chap. 3.

[5]Polehn, R. A., "Thermal Response of an Axisymmetric, Charring-Decomposing Polymer," Ph.D. Dissertation, Univ. of Tennessee at Knoxville, Knoxville, TN, in progress.

Effects of Nonequilibrium Chemistry on the Reference Temperature Method and Reynolds Analogy

James D. Ott* and John D. Anderson Jr.†
University of Maryland,
College Park, Maryland 20742

Nomenclature

M_e = Mach number at the edge
T = temperature, K

Subscripts
e = edge condition
F = quantity evaluated using correlation factor
w = wall condition

I. Introduction

IN the development of hypersonic vehicles, the accurate prediction of heat transfer and skin friction is required. One way to predict these values is by using a reference temperature method to find the skin friction, and then to use Reynold's analogy to calculate the heat transfer through the Stanton number. These methods were developed for nonreacting air and may not be accurate for nonequilibrium chemically reacting air. The purpose of this study is to determine what effect nonequilibrium air chemistry has on the reference temperature method and Reynold's analogy; then to correlate a modification to the reference temperature method to make

Received June 1, 1993; revision received Sept. 14, 1993; accepted for publication Sept. 30, 1993.

*Graduate Assistant, Hypersonic Group, Department of Aerospace Engineering. Student Member AIAA.

†Professor, Department of Aerospace Engineering. Fellow AIAA.

it more applicable in the presence of nonequilibrium dissociated air at hypersonic speeds.

The concept of a reference temperature was developed by Rubesin and Johnson[1] as a way to evaluate the mean fluid properties in a compressible boundary layer. Dorrance[2] showed that a reference temperature method can be derived from the laminar boundary-layer equations resulting in a formula that is very close to the one developed by Eckert.[3] Young and Janssen[4] developed a reference temperature method for use above Mach 5.6 which they show giving better results than Ref. 1. For equilibrium air Cohen[5,6] found that the reference temperature or enthalpy are not simple formulas. He found correlations for the density and density-viscosity product as functions of enthalpy and pressure. Debrestian[7] examined how well the reference temperature method worked for a nonequilibrium, chemically reacting binary gas of oxygen. He found that for Mach numbers greater than 5 that the Young-Janssen formula gave better results than the Rubesin-Johnson formula.

The reference temperature method is a way to correlate the incompressible flat plate skin friction result for use in a compressible flow, resulting in[8]

$$c_f^* = (0.664/\sqrt{Re_x^*}) \qquad Re_x^* = \frac{\rho^* u_e x}{\mu^*} \tag{1}$$

with the star * indicating quantities evaluated at the reference temperature. Using Sutherland's law and the equation of state, the above formula can be reduced to

$$c_f^* = \frac{0.664}{\sqrt{Re_x}}\sqrt{C^*} \qquad C^* = \left(\frac{T^*}{T_e}\right)^{1/2}\left(\frac{T_e + 111.0}{T^* + 111.0}\right) \tag{2}$$

where T^* is the reference temperature calculated by the Young-Janssen formula given by

$$\frac{T^*}{T_e} = 1.28 + 0.023M_e^2 + 0.58\left(\frac{T_w}{T_e} - 1\right) \tag{3}$$

Reynold's analogy is a method used to estimate the Stanton number from the skin friction coefficient. Reynolds[9] postulated a relation between shear stress and heat transfer by relating the shear stress through the skin friction coefficient and the heat flux through the Stanton number. Van Driest[10] and White[11] showed this to be approximately true for a laminar boundary layer. Cohen[5,6] evaluated the Prandtl number Pr at the reference enthalpy and showed that the Stanton number was within 4% of the correct value and was usually high. Debrestian[7] examined Reynold's analogy with the skin friction coefficient evaluated at the Young-Janssen reference temperature and found that for $Pr = 0.75$ all cases had the Stanton number within 4% from his code's value. Van Driest[10] showed that $2C_H/c_f$ was closely approximated as $Pr^{-2/3}$ so Reynold's analogy becomes

$$C_H = \tfrac{1}{2}c_f Pr^{-2/3} \tag{4}$$

From Debrestian,[7] the Prandtl number used was 0.75.

The purpose of the present study is to determine what effect nonequilibrium air chemistry has on the skin friction and Stanton Number, and to examine the applicability of the reference temperature method and Reynold's analogy for such conditions. The nonequilibrium laminar boundary-layer equations were solved on a flat plate geometry for various Mach numbers, altitudes, and wall-to-edge temperature ratios. A comparison between exact values calculated by the boundary-layer code and approximate values from the reference temperature method is carried out, and a correlation is presented for obtaining improved results from the reference temperature method and Reynold's analogy.

II. Solution Procedure

The two-dimensional laminar boundary-layer equations for a seven species nonequilibrium chemically reacting gas were used.[11] The boundary-layer equations were transformed to a (ξ, η) coordinate system using the Lees-Dorodnitsyn transformation. The resulting equations were numerically solved using an implicit Crank-Nicholson approach as developed by Blottner.[12-14] A chemically reacting mixture of perfect gases was assumed. A seven chemical equation rate model was used with the forward and backward rate constants obtained from Gupta et al.[15] The values of enthalpy, specific heat, viscosity, and frozen thermal conductivity for each species, and the binary diffusion coefficients were calculated by correlations.[15] The boundary-layer edge values were taken to be freestream values at the altitudes used in this study.[16] For each case, a fully catalytic constant temperature wall was used. For the velocity, the no-slip condition was maintained. To start the code, the self-similar, chemically frozen, thermally perfect boundary-layer profile was used.[17]

III. Results and Discussion

For this study, a range of Mach numbers, altitudes, and wall-to-edge temperature ratios were used. The Mach numbers ranged from 5 to 25, covering the altitudes of 50,000–250,000 ft. The wall temperatures varied from about 306 K to about 1950 K. From the above conditions, 49 cases were implemented on a 4-m-long flat plate.

A. Skin Friction Results

Equations (2) and (3) were used to find the skin friction coefficient for comparison to the values found from the boundary-layer code. Since the actual skin friction coefficient varies with distance, the parameter used for comparison was $c_f\sqrt{Re_x}$, since this is a more constant value for the entire plate.

Table 1 Skin friction results

Altitude = 150,000 ft						
Mach	T_w, K	$c_f\sqrt{Re_x}$	$c_f^*\sqrt{Re_x}$	% difference	$(c_f^*\sqrt{Re_x})_F$	% difference
10	400.60	0.5299	0.5353	+1.02	0.5318	+0.36
10	1201.80	0.5058	0.4953	−2.07	0.5157	+1.95
10	1869.46	0.4907	0.4701	−4.01	0.5022	+2.36
15	400.60	0.4722	0.4758	+0.76	0.4753	+0.65
15	1201.80	0.4726	0.4520	−4.37	0.4744	+0.39
15	1869.46	0.4647	0.4360	−6.18	0.4689	+0.91
20	400.60	0.4218	0.4279	+1.46	0.4293	+1.79
20	1201.80	0.4453	0.4133	−7.19	0.4365	−1.98
20	1869.46	0.4424	0.4028	−8.95	0.4361	−1.43
25	1201.80	0.4221	0.3808	−9.78	0.4041	−4.26
25	1869.46	0.4235	0.3737	−11.76	0.4066	−3.98

Table 2 Stanton number results

Altitude = 150,000 ft						
Mach	T_w, K	$C_H\sqrt{Re_x}$	$C_H^*\sqrt{Re_x}$	% difference	$(C_H^*\sqrt{Re_x})_F$	% difference
10	400.60	0.3412	0.3242	−4.98	0.3373	−1.15
10	1201.80	0.3275	0.3000	−8.38	0.3270	−0.13
10	1869.46	0.3173	0.2853	−10.10	0.3185	+0.38
15	400.60	0.3023	0.2882	−4.68	0.3014	−0.30
15	1201.80	0.2985	0.2737	−8.28	0.3009	+0.81
15	1869.46	0.2922	0.2641	−9.62	0.2974	+1.79
20	400.60	0.2803	0.2592	−7.53	0.2723	−2.86
20	1201.80	0.2801	0.2503	−10.63	0.2768	−1.17
20	1869.46	0.2766	0.2440	−11.81	0.2766	−0.02
25	1201.80	0.2663	0.2306	−13.38	0.2563	−3.75
25	1869.46	0.2649	0.2263	−14.54	0.2579	−2.63

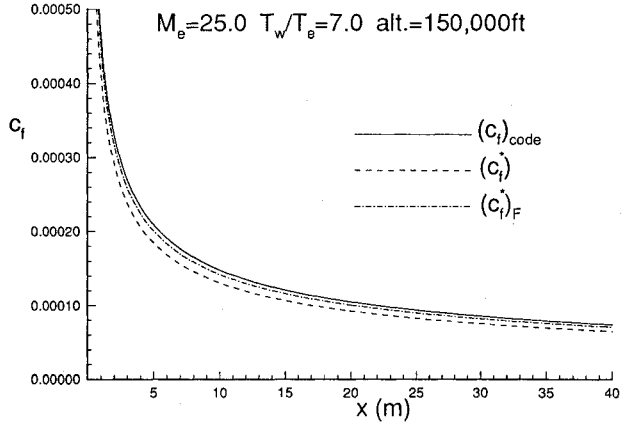

Fig. 1 Comparison of the variation of the skin friction coefficient on a 40-m flat plate for the code's value, the original reference temperature method's value, and the modified reference temperature method's value vs distance downstream.

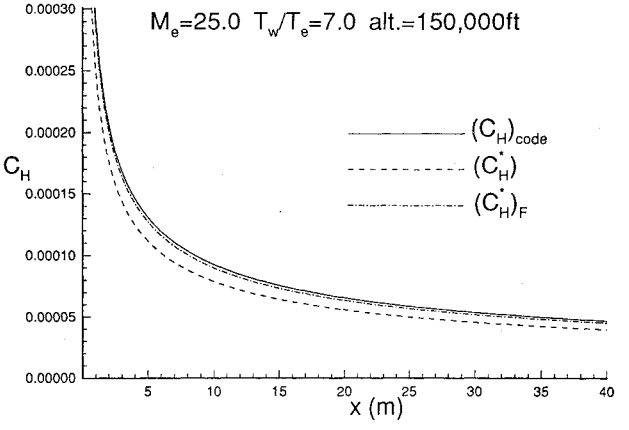

Fig. 2 Comparison of the variation of the Stanton number on a 40-m flat plate for the code's value, the original reference temperature method's value with a $Pr = 0.75$, and the modified reference temperature method's value with a $Pr = 0.70$ vs distance downstream.

For the cases done in this study it was found that the reference temperature method's skin friction coefficient varied between about 6% high for a low wall temperature and Mach number case, to about 12% low for a high Mach number and wall temperature case. Sample results for 150,000 ft are tabulated in Table 1 in columns 1–5. The exact coefficient value given is actually the average of the code's value over the 4 m.

A correlation was attempted to try to improve the results for the skin friction coefficient in the presence of nonequilibrium chemistry. The correlation modifies the value of T^* by multiplying the Young-Janssen formula by a factor F to provide a better temperature at which to evaluate the skin friction coefficient:

$$\left(\frac{T^*}{T_e}\right)_F = F\left[1.28 + 0.023M_e^2 + 0.58\left(\frac{T_w}{T_e} - 1\right)\right] \quad (5)$$

The correlation factor that was found and provided improved results was[17]

$$F = 7.494(M_e^{-0.067})[(T_e + T_w)^{-0.281}] \quad (6)$$

The coefficient (7.494) has units of temperature (K) to make the correlation factor nondimensional. In general, the modified reference temperature method generated better values for the skin friction coefficient than the original reference temperature method predicted. For Mach 5 and a low wall temperature, the correlation made the error worse, but only by a small amount. The modification brought the error to within ±5.5% of the code's value for all cases. Given in Table 1, in columns 6 and 7, are the skin friction coefficients generated with the modification and the error as compared to the code's values. Figure 1 compares the original and modified method's skin friction coefficients to the code's values. It can be seen that the modified method provides a better approximation than the original method.

B. Heat Transfer Results

Reynold's analogy, Eq. (4), was used to find the Stanton number to compare to the code's value. The skin friction coefficient was found by using Eq. (2), and the Prandtl number used was 0.75. From the cases done it was found that Reynold's analogy underpredicted the Stanton number for most cases, some by as much as 14% low. Then, the modified method was used to obtain the skin friction coefficient, and the Prandtl number was changed to 0.70 which more closely approximated the actual wall values which ranged from 0.68 to 0.71. The use of the modified method and $Pr = 0.70$ reduced the errors to within about 5.5% for all but a few cases. Sample results are given in Table 2. Figure 2 shows the variation of the Stanton number over a 40-m plate. The modified method is shown to more closely approximate the code's value.

IV. Conclusions

Calculations have been presented for the flow over a flat plate at zero angle of attack to address the effects of chemical nonequilibrium on skin friction and heat transfer. The major conclusions from this study are as follows:

1) The modified reference temperature method greatly reduced the error in the skin friction coefficient, especially for higher Mach numbers and wall temperatures.

2) The use of the modified method's skin friction coefficient and a $Pr = 0.70$ gave improved results in finding the Stanton number for higher Mach numbers and wall temperatures.

3) For a nonequilibrium flow, Reynold's analogy still holds, as long as good approximations for the skin friction coefficient and the governing Prandtl number are found.

Acknowledgments

This work was supported under Grant NAG-1-1192S-1, with Lawrence Huebner, Supersonic/Hypersonic Branch, NASA Langley, as technical monitor.

References

[1]Rubesin, M. W., and Johnson, H. A., "A Critical Review of Skin Friction and Heat Transfer Solutions of the Laminar Boundary Layer of a Flat Plate," *Transactions of the American Society of Mechanical Engineers*, Vol. 71, May 1949, pp. 383–388.

[2]Dorrance, W. H., *Viscous Hypersonic Flow*, McGraw-Hill, New York, 1962.

[3]Eckert, E. R. G., "Engineering Relations for Heat Transfer and Friction in High Velocity Laminar and Turbulent Boundary Layer Flow over Surfaces with Constant Pressure and Temperature," *Transactions of the American Society of Mechanical Engineers*, Vol. 78, No. 6, 1956, pp. 1273–1383.

[4]Young, G. B., and Janssen, E., "The Compressible Boundary Layer," *Journal of the Aeronautical Sciences*, Vol. 19, No. 4, 1952, pp. 229–236, 288.

[5]Cohen, N. B., "Correlation Formulas and Tables of Density and Some Transport Properties of Equilibrium Dissociated Air for Use in the Boundary Layer Equations," NASA TN D-194, Feb. 1960.

[6]Cohen, N. B., "Boundary Layer Similar Solutions and Correlation Equations for Laminar Heat Transfer Distribution in Equilibrium Air at Velocities up to 41,000 Feet Per Second," NASA TR R-118, 1961.

[7]Debrestian, D. J., "A Numerical Investigation of the Applicability of the Reference Temperature Method and Reynolds Analogy to Nonequilibrium Chemically Reacting Boundary Layers," M.S. Thesis, Univ. of Maryland, College Park, MD, 1992.

[8]Anderson, J. D., Jr., *Hypersonic and High Temperature Gas Dynamics*, McGraw-Hill, New York, 1989.

[9]Reynolds, O., *Proceedings of the Manchester Literary and Philosophical Society*, Vol. 14, King of Prussia, PA, 1874, pp. 7–12.

[10]Van Driest, E. R., "Investigations of Laminar Boundary Layers in Compressible Fluids Using the Crocco Method," NACA TN-2597, 1952.

[11]White, F. M., *Viscous Fluid Flow*, McGraw-Hill, New York, 1991.

[12]Blottner, F. G., "Non-Equilibrium Laminar Boundary Layer Flow of Ionized Air," General Electric TIS R64SD56, 1964; see also *AIAA Journal*, Vol. 2, No. 11, 1964, pp. 1921–1927.

[13]Blottner, F. G., "Finite Difference Methods of the Solution of the Boundary Layer Equations," *AIAA Journal*, Vol. 8, No. 2, 1970, pp. 193–205.

[14]Blottner, F. G., "Nonequilibrium Laminar Boundary Layer Flow of a Binary Gas," General Electric TIS R63SD17, 1963.

[15]Gupta, R. N., Yos, J. M., Thompson, R. A., and Lee, K. P., "A Review of Reaction Rates and Thermodynamic and Transport Properties for an 11-Species Air Model for Chemical and Thermal Nonequilibrium Calculations to 30,000 K," NASA Ref. Publ. 1232, Aug. 1990.

[16]*U.S. Standard Atmosphere 1976*, NOAA, NASA, USAF, Oct. 1976.

[17]Ott, J. D., "The Effects of Nonequilibrium Chemistry on the Reference Temperature Method and Reynolds Analogy," M.S. Thesis, Univ. of Maryland, College Park, MD, 1993.